DEVRE KURAMI-2 ÇALIŞMA SORULARI

1-) $\frac{d}{dt} i_L(t) = -\frac{1}{3} i_L(t) - \frac{1}{2} e(t) - \frac{1}{6} j_1(t) - \frac{1}{6} j_2(t) - \frac{1}{2}\frac{d}{dt} j_1(t)$;

şeklinde durum denklemi verilmiştir.

$i_L(0) = 5A$, $e(t) = 4V$, $j_1(t) = \sin t$ A ; $j_2(t) = e^{-t}$ A olduğuna göre

tam çözümü bulunuz?

Verilenleri yerine yazarsak denklem şöyle olur:

$$\dot{i}_L(t) + \frac{1}{3} i_L(t) = -2 - \frac{1}{6}\sin t - \frac{1}{2}\cos t - \frac{1}{6} e^{-t}$$

Öz çözüm

$$P + \frac{1}{3} = 0 \qquad P_1 = -\frac{1}{3}$$

$$i_{L_{öz}}(t) = C_1 . e^{-\frac{1}{3}t}$$

$$i_{L_{öz}}(0) = C_1 . e^{-\frac{1}{3}.0} = 5 \qquad C_1 = 5$$

$$\boxed{i_{L_{öz}}(t) = 5.e^{-\frac{1}{3}t}}$$

Özel çözüm

$$i_{özel}(t) = b_1 + b_2 \sin t + b_3 \cos t + b_4 e^{-t}$$

$$\dot{i}_{özel}(t) = b_2 \cos t - b_3 \sin t - b_4 e^{-t}$$

$$b_2 \cos t - b_3 \sin t - b_4 e^{-t} + \frac{1}{3} b_1 + \frac{1}{3} b_2 \sin t + \frac{1}{3} b_3 \cos t + \frac{1}{3} b_4 e^{-t} = -2 - \frac{1}{6}\sin t - \frac{1}{2}\cos t - \frac{1}{6} e^{-t}$$

$$b_2 + \frac{1}{3} b_3 = -\frac{1}{2} \qquad -b_3 + \frac{1}{3} b_2 = -\frac{1}{6} \qquad -b_4 + \frac{1}{3} b_4 = -\frac{1}{6} \qquad \frac{1}{3} b_1 = -2$$

$$b_1 = -6$$

$$-\frac{2 b_4}{\cancel{3}} = -\frac{1}{\cancel{6}} \qquad b_4 = \frac{1}{4}$$

$$3/\ 6b_2 + 2b_3 = -3$$
$$2b_2 - 6b_3 = -1$$
$$20 b_2 = -10$$
$$b_2 = -\frac{1}{2} \qquad b_3 = 0$$

$$\boxed{i_{L_{özel}}(t) = -6 - \frac{1}{2}\sin t + \frac{1}{4} e^{-t}}$$

$$i_{L_{Tam}} = C_1 . e^{-\frac{1}{3}t} - 6 - \frac{1}{2}\sin t + \frac{1}{4} e^{-t}$$

$$i_L(0) = C_1 - 6 - 0 + \frac{1}{4} = 5$$

$$C_1 = \frac{43}{4}$$

$$i_{L_{tam}}(t) = \frac{43}{4} e^{-\frac{1}{3}t} - 6 - \frac{1}{2}\sin t + \frac{1}{4} e^{-t}$$

4- $\dfrac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} \sin t \\ \cos t \end{bmatrix}$, $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Durum denklemi verilen devrede geçici ve kalıcı çözümden bahsedilebilirmi? Eğer bahsedilebilirse geçici ve kalıcı çözümleri bulunuz?

$|\lambda I - A| = 0$

$$\begin{vmatrix} \lambda & 0 \\ 0 & \lambda \end{vmatrix} - \begin{vmatrix} -2 & -1 \\ 1 & 0 \end{vmatrix} = \begin{vmatrix} \lambda+2 & 1 \\ -1 & \lambda \end{vmatrix}$$

$\lambda(\lambda+2) - (1.(-1)) = 0$

$\lambda^2 + 2\lambda + 1 = 0$

$\lambda_{1,2} = -1$

Homojen Çözüm

$x_{1h} = c_{11}.e^{-t} + c_{12}.t.e^{-t}$

$x_{2h} = c_{21}e^{-t} + c_{22}.t.e^{-t}$

$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_h = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}\begin{bmatrix} e^{-t} \\ te^{-t} \end{bmatrix}$

$\dot{X} = AX$ şeklinde yazılır.

$\dot{x}_{1h} = -c_{11}e^{-t} + c_{12}e^{-t} - c_{12}te^{-t}$

$\dot{x}_{2h} = -c_{21}e^{-t} + c_{22}e^{-t} - c_{22}t.e^{-t}$

$$\begin{bmatrix} -c_{11}+c_{12} & -c_{12} \\ -c_{21}+c_{22} & -c_{22} \end{bmatrix} \cdot \cancel{\begin{bmatrix} e^{-t} \\ t.e^{-t} \end{bmatrix}} = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \cdot \cancel{\begin{bmatrix} e^{-t} \\ t.e^{-t} \end{bmatrix}}$$

$-c_{11} + c_{12} = -2c_{11} - c_{21}$

$c_{11} + c_{12} = -c_{21}$

$-c_{12} = -2c_{12} - c_{22}$

$c_{12} = -c_{22}$

$-c_{21} + c_{22} = c_{11}$

$-c_{21} = c_{11} - c_{22}$

$-c_{22} = c_{12}$

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_h = \begin{bmatrix} c_{11} & -c_{22} \\ -c_{11}+c_{22} & c_{22} \end{bmatrix}\begin{bmatrix} e^{-t} \\ t.e^{-t} \end{bmatrix} \rightarrow t=0$ için $\begin{bmatrix} c_{11} & -c_{22} \\ -c_{11}+c_{22} & c_{22} \end{bmatrix}\cancel{\begin{bmatrix} 1 \\ 0 \end{bmatrix}} = \cancel{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}$

$c_{11} = 1 \qquad c_{22} = 1$

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{öz} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} e^{-t} \\ te^{-t} \end{bmatrix}}$$ öz çözüm.

Özel çözüm

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{\text{özel}} = \begin{bmatrix} A\cos t + B\sin t \\ C\cos t + D\sin t \end{bmatrix}$$

türevi alınıp denklemde yerine konur.

$$\begin{bmatrix} -A\sin t + B\cos t \\ -C\sin t + D\cos t \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} A\cos t + B\sin t \\ C\cos t + D\sin t \end{bmatrix} + \begin{bmatrix} \sin t \\ \cos t \end{bmatrix}$$

$$-A\sin t + B\cos t = -2A\cos t - 2B\sin t - C\cos t - D\sin t + \sin t$$

$$\sin t(\underbrace{-A+2B+D}_{1}) + \cos t(\underbrace{B+2A+C}_{0}) = \sin t$$

$$-C\sin t + D\cos t = A\cos t + B\sin t + 0 + \cos t$$

$$\sin t(\underbrace{-C-B}_{0}) + \cos t(\underbrace{D-A}_{1}) = \cos t$$

$C = -B$

$B+2A+C=0 \Rightarrow A=0$

$B=0$

$C=0$

$D=1$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{\text{özel}} = \begin{bmatrix} 0 \\ \sin t \end{bmatrix} \longrightarrow \text{özel çözüm.}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{\text{tam}} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{\text{öz}} + \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{\text{özel}}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{\text{tam}} = \underbrace{\begin{bmatrix} e^{-t} - te^{-t} \\ te^{-t} \end{bmatrix}}_{\text{geçici çözüm}} + \underbrace{\begin{bmatrix} 0 \\ \sin t \end{bmatrix}}_{\text{kalıcı çözüm}}$$

$t \to \infty$'a giderken

$e^{-t} \to 0$'a gider

(5-) Bir devrenin durum denklemi aşağıdaki gibi elde edilmiştir.

$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-t} \quad ; \quad \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Durum değişkenlerine ait tam çözümü bulunuz?

$$|\lambda I - A| = 0$$

$$\left| \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix} \right| = \begin{vmatrix} \lambda+2 & 0 \\ -1 & \lambda \end{vmatrix} \Rightarrow \lambda(\lambda+2) - (0.(-1)) = 0$$

$$\lambda(\lambda+2) = 0$$

$$\lambda_1 = 0 \quad \lambda_2 = -2$$

Homojen Çözüm

$$X_{1h} = C_{11} \cdot e^{-0.t} + C_{12} e^{-2t} \xrightarrow[\text{alalım.}]{\text{Türevlerini}} \dot{X}_{1h} = -2C_{12}e^{-2t}$$

$$X_{2h} = C_{21} e^{-0.t} + C_{22} \cdot e^{-2t} \qquad \dot{X}_{2h} = -2C_{22}e^{-2t}$$

$$\dot{X} = AX$$

$$\begin{bmatrix} 0 & -2C_{12} \\ 0 & -2C_{22} \end{bmatrix} \cancel{\begin{bmatrix} 1 \\ e^{-2t} \end{bmatrix}} = \begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \cancel{\begin{bmatrix} 1 \\ e^{-2t} \end{bmatrix}}$$

$$\begin{bmatrix} 0 & -2C_{12} \\ 0 & -2C_{22} \end{bmatrix} = \begin{bmatrix} -2C_{11} & -2C_{12} \\ C_{11} & C_{12} \end{bmatrix}$$

$$C_{11} = 0 \qquad -2C_{22} = C_{12}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_h = \begin{bmatrix} 0 & -2C_{22} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} 1 \\ e^{-2t} \end{bmatrix} \xrightarrow{t=0} \begin{bmatrix} 0 & -2C_{22} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$-2C_{22} = 1 \qquad C_{22} = -\frac{1}{2}$$

$$C_{21} + C_{22} = -1 \qquad C_{21} = -\frac{1}{2}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{öz} = \begin{bmatrix} 0 & 1 \\ -1/2 & -1/2 \end{bmatrix} \begin{bmatrix} 1 \\ e^{-2t} \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{öz} = \begin{bmatrix} e^{-2t} \\ -\frac{1}{2} - \frac{1}{2}e^{-2t} \end{bmatrix}$$

Özel çözüm

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}_{özel} = \begin{bmatrix} K_1\, e^{-t} \\ K_2 . e^{-t} \end{bmatrix} \Rightarrow$$

$$\dot{\underline{X}} = A\,\underline{X} + B(t)$$

$$\begin{bmatrix} -K_1\, e^{-t} \\ -K_2\, e^{-t} \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} K_1 . e^{-t} \\ K_2 . e^{-t} \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-t}$$

$$-K_1 e^{-t} = -2 K_1 e^{-t} - e^{-t} \qquad -K_2 e^{-t} = K_1 . e^{-t} + e^{-t}$$

$$K_1 e^{-t} = -e^{-t} \qquad -K_2 e^{-t} = 0$$

$$K_1 = -1 \qquad K_2 = 0$$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}_{özel} = \begin{bmatrix} -e^{-t} \\ 0 \end{bmatrix}$$

$$\left[X(t) \right]_{zor} = \left[X(t)_{hom.} + x(t)_{özel} \right]_{t=0} = 0$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}_{zor} = \begin{bmatrix} 0 & -2\overset{*}{C}_{22} \\ \overset{*}{C}_{21} & \overset{*}{C}_{22} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ e^{-2t} \end{bmatrix}_{t=0} + \begin{bmatrix} -\overset{-1}{e^{-t}} \\ 0 \end{bmatrix}_{t=0} = 0$$

$$0 - 2\overset{*}{C}_{22} - 1 = 0 \qquad \overset{*}{C}_{22} = -\frac{1}{2}$$

$$\overset{*}{C}_{21} + \overset{*}{C}_{22} + 0 = 0 \qquad \overset{*}{C}_{21} = \frac{1}{2}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{zor} = \begin{bmatrix} 0 & 1 \\ 1/2 & -1/2 \end{bmatrix} \begin{bmatrix} 1 \\ e^{-2t} \end{bmatrix} + \begin{bmatrix} -e^{-t} \\ 0 \end{bmatrix}$$

$$[x]_{tam} = [x]_{zor} + [x]_{özel}$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}_{tam} = \begin{bmatrix} 0 & 1 \\ 1/2 & -1/2 \end{bmatrix} \begin{bmatrix} 1 \\ e^{-2t} \end{bmatrix} + \begin{bmatrix} -e^{-t} \\ 0 \end{bmatrix} + \begin{bmatrix} -e^{-t} \\ 0 \end{bmatrix}$$

⑥ Birinci mertebeden bir devrenin durum denklemi;

$\frac{d}{dt} i_L(t) + 2\, i_L(t) = 4t+6$ olarak elde edilmiştir. Buna göre;

$i_L(0)=2A$

a-) Öz çözümü

b-) Zorlanmış çözümü

c-) Tam çözümü bulunuz.

d-) Kararlılığı inceleyiniz.

a-) $p+2=0$

$p=-2$

$i_L(t)_{öz} = c_1 . e^{-2t}$

$i_L(0)_{öz} = c_1 . e^{-2.0} = 2$

$C_1 = 2$ bulunur.

$\boxed{i_L(t)_{öz} = 2.e^{-2t}}$

b-) $i_L(t)_{özel} = b_1 + b_2 t$

$\dot{i}_L(t)_{özel} = b_2$

$\boxed{i_L(t)_{özel} = 2+2t}$

$b_2 + 2(b_1 + b_2 t) = 4t+6$

$b_2 + 2b_1 + 2b_2 t = 4t+6$

$2b_2 = 4 \qquad b_2 + 2b_1 = 6$

$\underline{b_2 = 2} \qquad \underline{b_1 = 2}$

$i_{L_{zor}} = \left[i_{L_h}(t) + i_{L_{özel}}(t) \right]_{t=0} = 0$

$\left[c_1 . e^{-2t} + 2 + \underbrace{2t}_{0} \right]_{t=0} = 0$

$c_1 + 2 = 0$

$c_1 = -2$

$\boxed{i_{L_{zor}}(t) = -2e^{-2t} + 2 + 2t}$

c-) $i_{L_{tam}}(t) = i_{L_{öz}}(t) + i_{L_{zor}}(t)$

$= 2.e^{-2t} + 2t + 2 - 2.e^{-2t}$

$i_{L_{tam}}(t) = 2t+2$

d-) $t \to \infty$ için $i_L(t) \to \infty$ gider. Bu nedenle devre kararsızdır.

7) $\frac{d}{dt}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}-2 & 0\\1 & -3\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}+\begin{bmatrix}1\\0\end{bmatrix}\sin t \quad , \quad \begin{bmatrix}x_1(0)\\x_2(0)\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$

denkleminin tam çözümünü bulunuz. ve sistemin kararlılığını inceleyiniz?

$|\lambda I - A| = 0$

$$\begin{vmatrix}\lambda & 0\\0 & \lambda\end{vmatrix}-\begin{vmatrix}-2 & 0\\1 & -3\end{vmatrix}=0 \Rightarrow \begin{vmatrix}\lambda+2 & 0\\-1 & \lambda+3\end{vmatrix}=0 \qquad (\lambda+2)(\lambda+3)-0.(-1)=0$$

$$\lambda_1=-2 \quad \lambda_2=-3$$

$$x_{1h}=c_{11}e^{-2t}+c_{12}e^{-3t}$$
$$x_{2h}=c_{21}e^{-2t}+c_{22}e^{-3t}$$

türevlerini alırsak $\Rightarrow$

$$\dot{x}_{1h}=-2c_{11}e^{-2t}-3c_{12}e^{-3t}$$
$$\dot{x}_{2h}=-2c_{21}e^{-2t}-3c_{22}e^{-3t}$$

$\underline{\dot{x}}=A\underline{x}$ denk. yerine koyalım.

$$\begin{bmatrix}-2c_{11}e^{-2t}-3c_{12}e^{-3t}\\-2c_{21}e^{-2t}-3c_{22}e^{-3t}\end{bmatrix}=\begin{bmatrix}-2 & 0\\1 & -3\end{bmatrix}\begin{bmatrix}c_{11}e^{-2t}+c_{12}e^{-3t}\\c_{21}e^{-2t}+c_{22}e^{-3t}\end{bmatrix}$$

$$\begin{bmatrix}-2c_{11} & -3c_{12}\\-2c_{21} & -3c_{22}\end{bmatrix}\cancel{\begin{bmatrix}e^{-2t}\\e^{-3t}\end{bmatrix}}=\begin{bmatrix}-2 & 0\\1 & -3\end{bmatrix}\begin{bmatrix}c_{11} & c_{12}\\c_{21} & c_{22}\end{bmatrix}\cancel{\begin{bmatrix}e^{-2t}\\e^{-3t}\end{bmatrix}}$$

$-2c_{11}=-2c_{11} \qquad -3c_{12}=-2c_{12} \qquad -2c_{21}=c_{11}-3c_{21} \qquad -3c_{22}=c_{12}-3c_{22}$

$c_{11}=c_{21} \qquad c_{12}=0$

$$\underbrace{\begin{bmatrix}x_1\\x_2\end{bmatrix}}_{\text{homojen}}=\begin{bmatrix}c_{21} & 0\\c_{21} & c_{22}\end{bmatrix}\begin{bmatrix}e^{-2t}\\e^{-3t}\end{bmatrix}$$

$$\begin{bmatrix}x_1\\x_2\end{bmatrix}_{öz}=\begin{bmatrix}c_{21} & 0\\c_{21} & c_{22}\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$$

t=0 için

$c_{21}=1$

$c_{21}+c_{22}=0$

$c_{22}=-1$

$$\boxed{\begin{bmatrix}x_1\\x_2\end{bmatrix}_{öz}=\begin{bmatrix}1 & 0\\1 & -1\end{bmatrix}\begin{bmatrix}e^{-2t}\\e^{-3t}\end{bmatrix}=\begin{bmatrix}e^{-2t}\\e^{-2t}-e^{-3t}\end{bmatrix}}$$

Özel çözüm

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{özel} = \begin{bmatrix} A\sin t + B\cos t \\ C\sin t + D\cos t \end{bmatrix}$$

$$\begin{bmatrix} A\cos t - B\sin t \\ C\cos t - D\sin t \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} A\sin t + B\cos t \\ C\sin t + D\cos t \end{bmatrix} + \begin{bmatrix} \sin t \\ 0 \end{bmatrix}$$

$$A\cos t - B\sin t = -2A\sin t - 2B\cos t + \sin t$$

$$\cos t\,(\underbrace{A+2B}_{0}) + \sin t\,(\underbrace{-B+2A}_{1}) = \sin t$$

$$A+2B=0$$
$$2/\ 2A-B=1$$
$$5A=2 \qquad A=\frac{2}{5} \qquad B=-\frac{1}{5}$$

$$C\cos t - D\sin t = A\sin t + B\cos t - 3C\sin t - 3D\cos t + 0$$

$$\cos t\,(\underbrace{C-B+3D}_{0}) + \sin t\,(\underbrace{-D-A+3C}_{0}) = 0$$

$$C+3D=-\frac{1}{5} \qquad 10C=1 \qquad C=\frac{1}{10}$$
$$3/\ 3C-D=\frac{2}{5} \qquad D=\frac{3}{10}-\frac{2}{5}=-\frac{1}{10}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{özel} = \begin{bmatrix} 2/5\sin t - 1/5\cos t \\ 1/10\sin t - 1/10\cos t \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{zor} = \underbrace{\begin{bmatrix} \overset{*}{c}_{21} & 0 \\ \overset{*}{c}_{21} & \overset{*}{c}_{22} \end{bmatrix} \begin{bmatrix} e^{-2t} \xrightarrow{} 1 \\ e^{-3t} \xrightarrow{} 1 \end{bmatrix}_{t=0}}_{homojen} + \underbrace{\begin{bmatrix} 2/5\sin t - 1/5\cos t \\ 1/10\sin t - 1/10\cos t \end{bmatrix}_{t=0}}_{özel} = 0$$

$$\overset{*}{c}_{21} = 1/5 \qquad \overset{*}{c}_{21} + \overset{*}{c}_{22} = \frac{1}{10} \Rightarrow \overset{*}{c}_{22} = -\frac{1}{10}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{tam} = \underbrace{\begin{bmatrix} 1/5 & 0 \\ 1/5 & -1/10 \end{bmatrix} \begin{bmatrix} e^{-2t} \\ e^{-3t} \end{bmatrix} + \begin{bmatrix} 2/5\sin t - 1/5\cos t \\ 1/10\sin t - 1/10\cos t \end{bmatrix}}_{X_{zor}} + \underbrace{\begin{bmatrix} e^{-2t} \\ e^{-2t} - e^{-3t} \end{bmatrix}}_{X_{öz}}$$

Devre asimptot kararlıdır.

(8) Bir devrenin durum denklemi aşağıdaki gibi verilmiştir.

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 2.\begin{bmatrix} 1 \\ -1 \end{bmatrix}\cos t \quad , \quad \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Tam çözümünü bulunuz? Devrenin kararlılığını inceleyiniz.

$$|\lambda I - A| = 0$$

$$\begin{vmatrix} \lambda & 0 \\ 0 & \lambda \end{vmatrix} - \begin{vmatrix} -2 & 0 \\ 1 & -2 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} \lambda+2 & 0 \\ -1 & \lambda+2 \end{vmatrix} = 0 \qquad (\lambda+2)^2 = 0$$

$$\lambda_{1,2} = -2$$

$$x_{1h} = A_1 . e^{-2t} + A_2 . t\, e^{-2t}$$
$$x_{2h} = B_1\, e^{-2t} + B_2 . t . e^{-2t}$$

$$\begin{bmatrix} x_{1h} \\ x_{2h} \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ B_1 & B_2 \end{bmatrix}\begin{bmatrix} e^{-2t} \\ t.e^{-2t} \end{bmatrix}$$

$$\dot{x}_{1h} = -2A_1\, e^{-2t} + A_2\, e^{-2t} - 2A_2 . t e^{-2t}$$
$$\dot{x}_{2h} = -2B_1\, e^{-2t} + B_2\, e^{-2t} - 2B_2 t . e^{-2t}$$

$$\begin{bmatrix} \dot{x}_{1h} \\ \dot{x}_{2h} \end{bmatrix} = \begin{bmatrix} -2A_1 + A_2 & -2A_2 \\ -2B_1 + B_2 & -2B_2 \end{bmatrix}\begin{bmatrix} e^{-2t} \\ t.e^{-2t} \end{bmatrix}$$

$\underline{\dot{X}} = A\underline{X}$ denkleminde yerine konursa;

$$\begin{bmatrix} -2A_1 + A_2 & -2A_2 \\ -2B_1 + B_2 & -2B_2 \end{bmatrix} . \cancel{\begin{bmatrix} e^{-2t} \\ t.e^{-2t} \end{bmatrix}} = \begin{bmatrix} -2 & 0 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} A_1 & A_2 \\ B_1 & B_2 \end{bmatrix}\cancel{\begin{bmatrix} e^{-2t} \\ t.e^{-2t} \end{bmatrix}}$$

$-2A_1 + A_2 = -2A_1$ $\quad$ $A_2 = 0$

$-2A_2 = -2A_2$

$-2B_1 + B_2 = A_1 - 2B_1$ $\quad$ $A_1 = B_2$

$-2B_2 = A_2 - 2B_2$ $\quad$ $A_2 = 0$

$$\begin{bmatrix} x_{1h} \\ x_{2h} \end{bmatrix} = \begin{bmatrix} B_2 & 0 \\ B_1 & B_2 \end{bmatrix} . \begin{bmatrix} e^{-2t} \\ t.e^{-2t} \end{bmatrix} \rightarrow \text{homojen çözüm.}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{öz} = \begin{bmatrix} B_2 & 0 \\ B_1 & B_2 \end{bmatrix}\begin{bmatrix} e^{-2t} \xrightarrow{} 1 \\ t.e^{-2t} \rightarrow 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \begin{matrix} B_2 = 1 \\ B_1 = 0 \end{matrix}$$

$t=0$ için

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{öz} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} e^{-2t} \\ t.e^{-2t} \end{bmatrix}} \rightarrow \text{öz çözüm.}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{öz} = \begin{bmatrix} e^{-2t} \\ t.e^{-2t} \end{bmatrix}$$

Özel çözüm

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}_{\text{özel}} = \begin{bmatrix} A\cos t + B\sin t \\ C\cos t + D\sin t \end{bmatrix}$$

türevi alınıp denklemde yerine konulur.

$$\begin{bmatrix} -A\sin t + B\cos t \\ -C\sin t + D\cos t \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} A\cos t + B\sin t \\ C\cos t + D\sin t \end{bmatrix} + \begin{bmatrix} 2\cos t \\ -2\cos t \end{bmatrix}$$

$$-A\sin t + B\cos t = -2A\cos t - 2B\sin t + 2\cos t$$

$$\sin t\,(\underbrace{-A+2B}_{0}) + \cos t\,(\underbrace{B+2A}_{2}) = 2\cos t \qquad \begin{aligned} 2/ -A+2B &= 0 \\ 2A+B &= 2 \\ \hline 5B &= 2 \end{aligned} \quad B = 2/5 \quad A = 4/5$$

$$-C\sin t + D\cos t = A\cos t + B\sin t - 2C\cos t - 2D\sin t - 2\cos t$$

$$\sin t\,(\underbrace{-C-B+2D}_{0}) + \cos t\,(\underbrace{D-A+2C}_{-2}) = -2\cos t \qquad \begin{aligned} 2/ 2D-C &= \tfrac{2}{5} \\ D+2C &= -\tfrac{6}{5} \\ \hline \end{aligned} \qquad \begin{aligned} 5D &= -\tfrac{2}{5} \\ D &= -\tfrac{2}{25} \\ C &= -\tfrac{14}{25} \end{aligned}$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}_{\text{özel}} = \begin{bmatrix} 4/5\cos t + 2/5\sin t \\ -14/25\cos t - 2/25\sin t \end{bmatrix} \longrightarrow \text{özel çözüm.}$$

Zorlanmış çözüm

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}_{\text{zor}} = \begin{bmatrix} \overset{*}{B}_2 & 0 \\ \overset{*}{B}_1 & \overset{*}{B}_2 \end{bmatrix} \begin{bmatrix} e^{-2t} \xrightarrow{} 1 \\ t.e^{-2t} \to 0 \end{bmatrix}_{t=0} + \begin{bmatrix} 4/5\cos t + 2/5\sin t \\ -14/25\cos t - 2/25\sin t \end{bmatrix}_{t=0 \text{ için}} = 0$$

$$\overset{*}{B}_2 + \frac{4}{5} = 0 \quad \overset{*}{B}_2 = -\frac{4}{5} \qquad \overset{*}{B}_1 - \frac{14}{25} = 0 \quad \overset{*}{B}_1 = \frac{14}{25}$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}_{\text{zor}} = \begin{bmatrix} -4/5 & 0 \\ 14/25 & -4/5 \end{bmatrix} \begin{bmatrix} e^{-2t} \\ t.e^{-2t} \end{bmatrix} + \begin{bmatrix} 4/5\cos t + 2/5\sin t \\ -14/25\cos t - 2/25\sin t \end{bmatrix}$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}_{\text{tam}} = \underbrace{\begin{bmatrix} e^{-2t} \\ t.e^{-2t} \end{bmatrix}}_{\text{öz}} + \underbrace{\begin{bmatrix} -4/5 & 0 \\ 14/25 & -4/5 \end{bmatrix} \begin{bmatrix} e^{-2t} \\ t.e^{-2t} \end{bmatrix} + \begin{bmatrix} 4/5\cos t + 2/5\sin t \\ -14/25\cos t - 2/25\sin t \end{bmatrix}}_{\text{zorlanmış}}$$

Devre asimptotik kararlıdır.

⑨ Bir devrenin durum denklemi aşağıdaki biçimde verilmiştir.

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 2.e^{-2t} \\ 5\cos t \end{bmatrix} , \quad \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Buna göre devrenin tam çözümünü bulunuz ve kararlılığını inceleyiniz?

$$|\lambda I - A| = 0$$

$$\begin{vmatrix} \lambda & 0 \\ 0 & \lambda \end{vmatrix} - \begin{vmatrix} 0 & -1 \\ 1 & -2 \end{vmatrix} = 0 \qquad \begin{vmatrix} \lambda & 1 \\ -1 & \lambda+2 \end{vmatrix} = 0 \qquad \begin{aligned} &\lambda(\lambda+2) - (1).(-1) = 0 \\ &\lambda^2 + 2\lambda + 1 = 0 \\ &(\lambda+1)^2 = 0 \\ &\lambda_{1,2} = -1 \end{aligned}$$

$$\begin{aligned} x_{1h} &= C_{11}.e^{-t} + C_{12}.t.e^{-t} \\ x_{2h} &= C_{21}\,e^{-t} + C_{22}.t.e^{-t} \end{aligned} \Rightarrow \begin{bmatrix} x_{1h} \\ x_{2h} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}\begin{bmatrix} e^{-t} \\ t.e^{-t} \end{bmatrix}$$

$$\begin{aligned} \dot{x}_{1h} &= -C_{11}e^{-t} + C_{12}.e^{-t} - C_{12}\,t\,e^{-t} \\ \dot{x}_{2h} &= -C_{21}e^{-t} + C_{22}e^{-t} - C_{22}.t.e^{-t} \end{aligned} \Rightarrow \begin{bmatrix} \dot{x}_{1h} \\ \dot{x}_{2h} \end{bmatrix} = \begin{bmatrix} -C_{11}+C_{12} & -C_{12} \\ -C_{21}+C_{22} & -C_{22} \end{bmatrix}\begin{bmatrix} e^{-t} \\ t.e^{-t} \end{bmatrix}$$

$\underline{\dot{X}} = A\,\underline{X}$ denkleminde yerine koyarsak.

$$\begin{bmatrix} -C_{11}+C_{12} & -C_{12} \\ -C_{21}+C_{22} & -C_{22} \end{bmatrix}\cancel{\begin{bmatrix} e^{-t} \\ t.e^{-t} \end{bmatrix}} = \begin{bmatrix} 0 & -1 \\ 1 & -2 \end{bmatrix}.\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}\cancel{\begin{bmatrix} e^{-t} \\ t.e^{-t} \end{bmatrix}}$$

$$-C_{11} + C_{12} = -C_{21} \qquad -C_{12} = -C_{22} \qquad \begin{aligned} -C_{21} + C_{22} &= C_{11} - 2C_{21} \\ C_{21} + C_{22} &= C_{11} \end{aligned} \qquad \begin{aligned} -C_{22} &= C_{12} - 2C_{22} \\ C_{22} &= C_{12} \end{aligned}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_h = \begin{bmatrix} C_{21}+C_{22} & C_{22} \\ C_{21} & C_{22} \end{bmatrix}\begin{bmatrix} e^{-t} \\ t.e^{-t} \end{bmatrix} \rightarrow \text{homojen çözüm}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{öz} = \begin{bmatrix} C_{21}+C_{22} & C_{22} \\ C_{21} & C_{22} \end{bmatrix}\begin{bmatrix} e^{-t} \rightarrow 1 \\ t.e^{-t} \rightarrow 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \begin{aligned} C_{21} + C_{22} &= 1 \\ C_{22} &= 0 \end{aligned} \qquad C_{21} = 1$$

$t=0$ için

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{öz} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} e^{-t} \\ t.e^{-t} \end{bmatrix} = \begin{bmatrix} e^{-t} \\ e^{-t} \end{bmatrix} \rightarrow \text{öz çözüm}$$

Özel çözüm

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{özel} = \begin{bmatrix} A_1 e^{-2t} + B_1 \cos t + C_1 \sin t \\ A_2 e^{-2t} + B_2 \cos t + C_2 \sin t \end{bmatrix}$$

türevini alıp denklemde yerine koyarsak;

$$\begin{bmatrix} -2A_1 . e^{-2t} - B_1 . \sin t + C_1 \cos t \\ -2A_2 e^{-2t} - B_2 . \sin t + C_2 \cos t \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} A_1 e^{-2t} + B_1 \cos t + C_1 \sin t \\ A_2 e^{-2t} + B_2 \cos t + C_2 \sin t \end{bmatrix} + \begin{bmatrix} 2e^{-2t} + 5\cos t \\ -5 \cos t \end{bmatrix}$$

$$-2A_1 e^{-2t} - B_1 \sin t + C_1 \cos t = -A_2 e^{-2t} - B_2 \cos t - C_2 \sin t + 2e^{-2t} + 5\cos t$$

$$e^{-2t}(\underbrace{-2A_1 + A_2}_{2}) + \sin t\,(\underbrace{-B_1 + C_2}_{0}) + \cos t\,(\underbrace{C_1 + B_2}_{5}) = 2e^{-2t} + 5\cos t$$

$-2A_1 + A_2 = 2$ $\quad B_1 = C_2$ $\quad C_1 + B_2 = 5$

$$-2A_2 e^{-2t} - B_2 \sin t + C_2 \cos t = A_1 e^{-2t} + B_1 \cos t + C_1 \sin t - 2A_2 e^{-2t} - 2B_2 \cos t - 2C_2 \sin t - 5\cos t$$

$$e^{-2t}(\underbrace{-\cancel{2A_2} - A_1 + \cancel{2A_2}}_{0}) + \sin t\,(\underbrace{-B_2 - C_1 + 2C_2}_{0}) + \cos t\,(\underbrace{C_2 - B_1 + 2B_2}_{-5}) = -5\cos t$$

$A_1 = 0$ $\quad 2C_2 - C_1 = B_2$ $\quad \cancel{\frac{5}{2}} - \cancel{\frac{5}{2}} + 2B_2 = -5$

$A_2 = 2$ $\quad 2C_2 - \cancel{C_1} = 5 - \cancel{C_1}$ $\quad B_2 = -\frac{5}{2}$

$B_1 = \frac{5}{2}$ $\quad C_2 = \frac{5}{2}$

$B_2 = -\frac{5}{2}$ $\quad C_1 = 5 + \frac{5}{2} = \frac{15}{2}$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{özel} = \begin{bmatrix} 5/2 \cos t + 15/2 \sin t \\ 2e^{-2t} - 5/2 \cos t + 5/2 \sin t \end{bmatrix} \rightarrow \text{özel çözüm}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{zor} = \begin{bmatrix} \overset{*}{C}_{21} + \overset{*}{C}_{22} & \overset{*}{C}_{22} \\ \overset{*}{C}_{21} & \overset{*}{C}_{22} \end{bmatrix} \begin{bmatrix} e^{-t} \xrightarrow{} 1 \\ t . e^{-t} \xrightarrow{} 0 \end{bmatrix}_{t=0} + \begin{bmatrix} 5/2 \cos t + 15/2 \sin t \\ 2e^{-2t} - 5/2 \cos t + 5/2 \sin t \end{bmatrix}_{t=0} = 0$$

$\overset{*}{C}_{21} + \overset{*}{C}_{22} = -\frac{5}{2}$ $\quad \overset{*}{C}_{21} = -2 + \frac{5}{2}$ $\quad \overset{*}{C}_{21} = \frac{1}{2}$ $\quad \overset{*}{C}_{22} = -3$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{tam} = \underbrace{\begin{bmatrix} e^{-t} \\ e^{-t} \end{bmatrix}}_{öz} + \underbrace{\begin{bmatrix} -5/2 & -3 \\ 1/2 & -3 \end{bmatrix} \begin{bmatrix} e^{-t} \\ t . e^{-t} \end{bmatrix} + \begin{bmatrix} 5/2 \cos t + 15/2 \sin t \\ 2e^{-2t} - 5/2 \cos t + 5/2 \sin t \end{bmatrix}}_{zorlanmış}$$

Devre asimptot kararlıdır.

(10) 2. dereceden bir devrenin durum denklemi aşağıdaki gibi verilmiştir.

$$\frac{d}{dt}\begin{bmatrix} V_{C_1}(t) \\ V_{C_2}(t) \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} V_{C_1}(t) \\ V_{C_2}(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \sin t \quad , \quad \begin{bmatrix} V_{C_1}(0) \\ V_{C_2}(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Buna göre devrenin öz çözümünü bulunuz. C_1 kapasitesine ait öz çözümün değişimini çiziniz?

$$|\lambda I - A| = 0$$

$$\left|\begin{matrix} \lambda & 0 \\ 0 & \lambda \end{matrix}\right| - \left|\begin{matrix} -2 & -1 \\ 1 & -2 \end{matrix}\right| = 0 \qquad \left|\begin{matrix} \lambda+2 & 1 \\ -1 & \lambda+2 \end{matrix}\right| = 0 \qquad (\lambda+2)^2 - (1)(-1) = 0$$

$$\lambda^2 + 4\lambda + 5 = 0$$

$$b^2 - 4ac = 16 - 4.1.5 = -4$$

$$\lambda_{1,2} = \frac{-4 \mp \sqrt{-4}}{2} = -2 \mp j$$

$$V_{C_{1h}}(t) = C_1 . e^{(-2-j)t} + \overset{*}{C_1} . e^{(-2+j)t}$$

$$= (A - jB)\, e^{(-2-j)t} + (A + jB) . e^{(-2+j)t}$$

$$= (A - jB)\, e^{-2t} . e^{-jt} + (A + jB) . e^{-2t} . e^{jt}$$

$$= e^{-2t}\Big((A - jB)(\cos t - j \sin t) + (A + jB) . (\cos t + j \sin t) \Big)$$

$$= e^{-2t}\big(A\cos t - \cancel{Aj\sin t} - \cancel{jB\cos t} - B\sin t + A\cos t + \cancel{Aj\sin t} + \cancel{jB\cos t} - B\sin t \big)$$

$$= e^{-2t}(2A \cos t - 2B \sin t)$$

$$V_{C_{2h}}(t) = C_2\, e^{(-2-j)t} + \overset{*}{C_2}\, e^{(-2+j)t}$$

$$= (C - jD) . e^{-2t} . e^{-jt} + (C + jD)\, e^{-2t} e^{jt}$$

$$= e^{-2t}\Big((C - jD)(\cos t - j\sin t) + (C + jD)(\cos t + j \sin t) \Big)$$

$$= e^{-2t}(2C \cos t - 2D \sin t)$$

$$\dot{V}_{C_{1h}}(t) = -2\, e^{-2t}(2A\cos t - 2B \sin t) + e^{-2t}(-2A \sin t - 2B \cos t)$$

$$\dot{V}_{C_{1h}}(t) = e^{-2t}\big((-4A - 2B)\cos t - (-4B + 2A) \sin t \big)$$

$$\dot{V}_{C_{2h}}(t) = -2\, e^{-2t}(2C \cos t - 2D \sin t) + e^{-2t}(-2C \sin t - 2D \cos t)$$

$$\dot{V}_{C_{2h}}(t) = e^{-2t}\big((-4C - 2D) \cos t - (-4D + 2C) \sin t \big)$$

$\underline{\dot{X}} = A\,\underline{X}$ denklemine konulursa;

$$\begin{bmatrix} e^{-2t}((-4A-2B)\cos t-(-4B+2A)\sin t) \\ e^{-2t}((-4C-2D)\cos t-(-4D+2C)\sin t) \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} e^{-2t}(2A\cos t-2B\sin t) \\ e^{-2t}(2C\cos t-2D\sin t) \end{bmatrix}$$

$$\cancel{e^{-2t}}((-4A-2B)\cos t-(-4B+2A)\sin t) = -2\cancel{e^{-2t}}(2A\cos t-2B\sin t)-\cancel{e^{-2t}}(2C\cos t-2D\sin t)$$

$$(-4A-2B)\cos t+(4B-2A)\sin t = (-4A-2C)\cos t+(-4B+2D)\sin t$$

$$-2B=-2C \qquad 4B-2A=-4B+2D$$
$$B=C \qquad 8B=2A+2D$$

$$(-4C-2D)\cos t+(4D-2C)\sin t = 2A\cos t-2B\sin t-4C\cos t+4D\sin t$$

$$(-4C-2D)\cos t+(4D-2C)\sin t = (2A-4C)\cos t+(-2B+4D)\sin t$$

$$4C-2D=2A-4C$$
$$8C=2A+2D$$

$$\begin{bmatrix} V_{C_{1h}}(t) \\ V_{C_{2h}}(t) \end{bmatrix} = \begin{bmatrix} 2A & -2B \\ 2C & -2D \end{bmatrix} \cdot \begin{bmatrix} \cos t \\ \sin t \end{bmatrix} e^{-2t}$$

$$\begin{bmatrix} V_{C_{1h}}(t) \\ V_{C_{2h}}(t) \end{bmatrix} = \begin{bmatrix} 8C-2D & -2C \\ 2C & -2D \end{bmatrix} \begin{bmatrix} \cos t \\ \sin t \end{bmatrix} e^{-2t} \rightarrow \text{homojen çözüm}$$

$$\begin{bmatrix} V_{C_1}(t) \\ V_{C_2}(t) \end{bmatrix}_{öz} = \begin{bmatrix} 8C-2D & -2C \\ 2C & -2D \end{bmatrix} \begin{bmatrix} \overset{\nearrow 1}{\cos t} \\ \underset{\searrow 0}{\sin t} \end{bmatrix} \overset{\nearrow 1}{e^{-2t}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$t=0$ için

$$8C-2D=1 \qquad 2C=0$$
$$D=-\frac{1}{2} \qquad C=0$$

$$\begin{bmatrix} V_{C_1}(t) \\ V_{C_2}(t) \end{bmatrix}_{öz} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos t \\ \sin t \end{bmatrix} e^{-2t} \rightarrow \text{öz çözüm}$$

$$\begin{bmatrix} V_{C_1}(t) \\ V_{C_2}(t) \end{bmatrix}_{öz} = \begin{bmatrix} e^{-2t}\cos t \\ e^{-2t}\sin t \end{bmatrix}$$

(11) Bir devrenin durum denklemi aşağıdaki gibi verilmiştir. İlk koşulları verildiğine göre öz çözümü bulunuz?

$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} \cos 3t \\ e^{-4t} \end{bmatrix} \quad ; \quad \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|\lambda I - A| = 0$$

$$\begin{vmatrix} \lambda & 0 \\ 0 & \lambda \end{vmatrix} - \begin{vmatrix} -2 & -1 \\ 1 & -2 \end{vmatrix} = 0 \qquad \begin{vmatrix} \lambda+2 & 1 \\ -1 & \lambda+2 \end{vmatrix} = 0 \qquad (\lambda+2)^2 - (1)(-1) = 0$$

$$\lambda^2 + 4\lambda + 5 = 0$$

$$\lambda_{1,2} = -2 \mp j$$

$$\begin{aligned} x_{1h}(t) &= c_1 . e^{(-2-j)t} + \overset{*}{c_1} . e^{(-2+j)t} \\ &= (A - jB) e^{-2t} . e^{-jt} + (A + jB) e^{-2t} . e^{jt} \\ &= e^{-2t} \left((A-jB)(\cos t - j \sin t) + (A+jB)(\cos t + j \sin t) \right) \\ &= e^{-2t} (2A \cos t - 2B \sin t) \end{aligned}$$

$$\begin{aligned} x_{2h}(t) &= c_2 \, e^{(-2-j)t} + \overset{*}{c_2} \, e^{(-2+j)t} \\ &= (C - jD) e^{-2t} . e^{-jt} + (C + jD) e^{-2t} . e^{jt} \\ &= e^{-2t} \left((C-jD)(\cos t - j \sin t) + (C+jB)(\cos t + j \sin t) \right) \\ &= e^{-2t} (2C \cos t - 2D \sin t) \end{aligned}$$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}_h = \begin{bmatrix} 2A & -2B \\ 2C & -2D \end{bmatrix} \begin{bmatrix} \cos t \\ \sin t \end{bmatrix} e^{-2t}$$

$$\begin{aligned} \dot{x}_{1h}(t) &= -2 e^{-2t} (2A \cos t - 2B \sin t) + e^{-2t} (-2A \sin t - 2B \cos t) \\ & e^{-2t} \left((-4A - 2B) \cos t + (4B - 2A) \sin t \right) \end{aligned}$$

$$\begin{aligned} \dot{x}_{2h}(t) &= -2 e^{-2t} (2C \cos t - 2D \sin t) + e^{-2t} (-2C \sin t - 2D \cos t) \\ & e^{-2t} \left((-4C - 2D) \cos t + (4D - 2C) \sin t \right) \end{aligned}$$

$\dot{\underline{X}} = A\underline{X}$ denkleminde yerine konulursa;

$$\begin{bmatrix} -4A-2B & 4B-2A \\ -4C-2D & 4D-2C \end{bmatrix} \cdot \cancel{\begin{bmatrix} \cos t \\ \sin t \end{bmatrix}} \cdot \cancel{e^{-2t}} = \begin{bmatrix} -2 & -1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 2A & -2B \\ 2C & -2D \end{bmatrix} \cancel{\begin{bmatrix} \cos t \\ \sin t \end{bmatrix}} \cancel{e^{-2t}}$$

$-4A-2B = -4A-2C$

$B = C$

$4B-2A = +4B+2D$

$A = -D$

$-4C-2D = 2A-4C$

$-D = A$

$4D-2C = -2B+4D$

$C = B$

$$\begin{bmatrix} x_{1h} \\ x_{2h} \end{bmatrix} = \begin{bmatrix} -2D & -2C \\ 2C & -2D \end{bmatrix} \begin{bmatrix} \cos t \\ \sin t \end{bmatrix} e^{-2t} \rightarrow \text{homojen çözüm}$$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}_{öz} = \begin{bmatrix} -2D & -2C \\ 2C & -2D \end{bmatrix} \begin{bmatrix} \overset{\nearrow 1}{\cos t} \\ \underset{\searrow 0}{\sin t} \end{bmatrix} \overset{\nearrow 1}{e^{-2t}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad t=0 \text{ için}$$

$-2D = 1$

$D = -\frac{1}{2}$

$2C = 0$

$C = 0$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}_{öz} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos t \\ \sin t \end{bmatrix} e^{-2t} = \begin{bmatrix} e^{-2t} \cos t \\ e^{-2t} \sin t \end{bmatrix} \rightarrow \text{öz çözümdür.}$$